Leverage, asymmetry and heavy tails in the high-dimensional factor stochastic volatility model

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Abstract
We propose a flexible high-dimensional factor stochastic volatility (SV) model with leverage effect based on the generalised hyperbolic skew Student’s t-distribution with heavy tails. With leverage, the model leads to different parametrisation forms, and thus is able to capture asymmetric leverage effect and skewness from asset-specific views. We also develop a highly efficient Markov chain Monte Carlo estimation procedure to analyse the univariate version of the model based on efficient importance sampling. With marginalisation of factors, extension to high-dimensional factor model is achieved with computational complexity shown to be linear in the number of factors and assets.

Introduction
In the literature of univariate SV models, parameter-driven SV models and observation-driven (G)ARCH type of models (Kim et al., 2018) are mainly used to model the following “stylised facts” of asset returns:

- Volatility clustering (consecutive volatile periods);
- Heavy tails of asset returns (extreme events);
- Asymmetry or skewness of asset returns (loss bigger than gains);
- Leverage effect (loss correlates with higher volatility).

When modelling a portfolio of asset returns, “stylised facts” of individual asset returns are linked via the hypothetical market portfolio. This makes modelling high-dimensional portfolios exponentially complex due to the “curse of dimensionality”.

Factor SV model
A straightforward remedy is through a factor model. A factor model naturally delivers systematic interpretation for the multivariate dynamics of a vector of time series, which can help track the sources of observed “stylised facts”. In the spirit of Chib et al. (2006), we propose the following factor SV model:

\[ y_{it} = \beta_{it} + \epsilon_{it}, \quad t = 1, \ldots, T. \]

\[ \{y_{it}\}_{t=1}^{T} \sim \text{Model (1)}, \quad \{\epsilon_{it}\}_{t=1}^{T} \sim \text{GARCH}(1,1). \]

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\[ W_{i1:T} \sim \text{GARCH}(1,1). \]

Figure 1: Different density shapes of generalised hyperbolic skew Student’s t-distribution. Left: varying with $\alpha \in [0, 1]$; Right: varying with $\beta = -2$.

MCMC sampler for the factor SV model
The sampling scheme is straightforward, and with marginalisation of factors, the loadings $\beta_{it}$ can be sampled efficiently. Notice the factor SV model process:

\[ \lambda_{i1:T} \sim \prod_{t=1}^{T} \lambda_{i1:t}^{(p)} \lambda_{i1:t}^{(g)}, \quad \lambda_{i1:t}^{(p)} \sim \text{GARCH}(1,1), \quad \lambda_{i1:t}^{(g)} \sim \text{GARCH}(1,1). \]

The MCMC sampler iterates over

- Sample $\lambda_{i1:T}$.
- Marginalise $f_{i1:T} = f_{i1:T}^{(p)} f_{i1:T}^{(g)}$.
- Sample $\lambda_{i1:t}^{(p)}$.
- Obtain $\lambda_{i1:T}^{(p)}$.
- Sample SV $\lambda_{i1:T}$ and mixture series $W_{i1:T}$ and $Q_{i1:T}$ as well as other hyperparameters $\alpha$ using sampling scheme for Model (1).

MCMC sampler for the univariate SV model
The last step of the sampler for the factor SV model is to apply an MCMC sampler for the $p = n$ individual univariate SV model (1). Thus our model is linearly scalable in number of assets $n$ and number of factors $p$. The MCMC sampler has two parts:

1. Sampling $\beta_{it}, y_{it}, W_{i1:T}$ and $\epsilon_{it}$.
2. Sampling $\lambda_{i1:T}^{(g)}$ and $W_{i1:T}$.

PGAS-ESS sampler
We term our sampler particle Gibbs with ancestor sampling using efficient importance sampling (ESS-PGAS). ESS stems from particle efficient importance sampling (PERIS) of Schardin and Kohu (2010) which builds a globally optimal importance density. Based on the fact that exponential family kernels are closed under multiplication, a sequential Monte Carlo sampler with EIS is given by

\[ \hat{b}_t, \hat{H}_t \mid (y_{1:t-1}, y_t) = \lambda_t (\sum_{i=1}^{M} \hat{b}_i + \epsilon_{it}), \]

\[ \hat{H}_t^n = \frac{\epsilon_{it} - \mu_H (\epsilon_{it} - \hat{H}_t^n + \epsilon_{it} - \hat{H}_t^n)}{\epsilon_{it} - \hat{H}_t^n}, \]

where $\epsilon_{it} = -\log(\epsilon_{it})$, $\mu_H = 1$ and $\lambda_t$ are importance parameters determined by a sequence of simple OLS, minimising the $\chi^2$-divergence between the importance density and the conditional posterior.

We can calculate the implied time-varying correlation as

\[ \text{Cov}_{it} = \sum_{j=1}^{k} \lambda_{ij} \phi_j (\epsilon_{it}). \]

Figure 5: Posterior mean of factor and stochastic volatility process. Left: SV $\epsilon_{it} | y_{1:t-1}$ factor; Middle: volatility $s_{it}$ of three chosen assets; Right: Impulse time-varying correlations $\text{Cov}_{it}$ among the three asset returns.

In the year of financial crisis the three correlation series start climbing up, one of which even shoots up to over 0.4. Yet outside the crisis period the correlation can be low in absolute value. So equicorrelation models are suboptimal in diversification.

Dynamic portfolio management
The solution of this MVP problem is given by $w_{it} = \frac{1}{\lambda_t} \hat{b}_t (y_{it} + \epsilon_{it})$. So, the value-at-risk (VaR) is given by

\[ w_{it} \hat{b}_t (y_{it} + \epsilon_{it}). \]

Sharp ratio and information ratio are defined similarly using filtered estimate of covariance matrix.

Table 1: Quality of VAR estimation using the US portfolio

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The table shows the average weekly MVP returns and variances for the US portfolios. Sharpe ratio is also calculated. Coloured dots indicate rejection of variance ratio at 5% level.

Table 2: The table shows the average weekly MVP returns and variances for the US portfolios. Sharpe ratio is also calculated. Coloured dots indicate rejection of variance ratio at 5% level.

Figure 6: The table shows the average weekly MVP returns and variances for the US portfolios. Sharpe ratio is also calculated. Coloured dots indicate rejection of variance ratio at 5% level.

Conclusion
We propose a high-dimensional factor SV model with leverage effect using the generalised hyperbolic skew Student’s t-distribution to address asymmetry and heavy tails of equity returns, as well as a highly efficient MCMC algorithm for Bayesian inference. The model is shown to be flexible enough to distinguish asset-specific mean and volatility dynamics from common factors. With shrinkage, the model helps answer whether leverage effect and return asymmetry are systematic or idiosyncratic.

References